PHYSICS

Does a Moving Body appear Cool?

THE question, "Does a moving body appear cool?", which I raised in an earlier note, has now received affirmative answers[1-3], in agreement with accepted theory[4]. These answers appeal to the Doppler effect[2], and implicitly to the equipartition theorem[1,3]. In this note a continued reserve towards accepted theory is maintained by attempting to show that neither line of argument enables one to discriminate between accepted theory and the view that temperature can be Lorentz-invariant.

Inertial frames will be taken in "standard configuration"[5]; quantities measured in a centre-of-mass frame I_0 will be given the suffix 0, and $\beta(u) \equiv [1-u^2/c^2]^{-1/2}$.

(*A*) *Ambiguities by arguing from the transverse Doppler effect.* The mean occupation number of a radiation mode j is the same in the rest frame of a black-body enclosure (n_{j0}) as it is in any other inertial frame (n_j). Thus if radiation is observed in a direction transverse to the direction of motion

$$n_{j0} = n_j \text{ and } n_{j0} = f(h\nu_0/kT_0) = f(\beta h\nu/kT_0) \qquad (1)$$

Here f denotes the Bose distribution function. Now one can argue either

$$T = T_0/\beta, \text{ that is } n_j = f(h\nu/kT) \qquad (2)$$

or

$$T = T_0, \text{ that is } n_j = f(\beta h\nu/kT) \qquad (3)$$

Equation (2) agrees with orthodox theory and associates with the moving radiation a distribution of rest-frame form. No general principle exists, however, which demands this. Thus equation (3) agrees with my theory[6] and implies the generalized distributions proposed in the appendix of the statistical mechanical version[7] of this theory. Thus the transverse Doppler effect cannot lead from equation (1) to a method of discriminating between the two theories on theoretical grounds. It suggests, however, an experimental method of distinguishing between them.

(*B*) *Experimental test.* Orthodox theory leads from Wien's displacement law $\lambda_{\max,0}T_0 = a$ ($a = 0{\cdot}29$ cm-degree) and $\lambda_{\max} = \beta\lambda_{\max,0}$ to $\lambda_{\max}T_0/\beta = a$. Assumption (2) yields a displacement law for a moving radiation enclosure which has the rest-frame form

$$\lambda_{\max}T = a \qquad (4)$$

Assumption (3), however, generalizes it to

$$\lambda_{\max}T/\beta = a \qquad (5)$$

The difference between equations (4) and (5) could yield an observational test if $\lambda_{\max}$ can be determined from the spectrum of a moving black body observed in I, and if (which seems more difficult) its temperature in I can be properly defined and determined independently. The product $\lambda_{\max}T$ must then be compared with the two theoretical values a and βa.

(*C*) *The general case against the orthodox transformation.* Because only temperature in special relativity is being considered, one would like to simplify and to clarify Noerdlinger's discussion. This can be done by considering two equilibrium systems A and B which are large enough to have parts of their surfaces near each other for an indefinite period, even though they are in uniform relative motion. Heat transfer can then take place at right angles to the motion by exchange of radiation between small areas in proximity (by appropriately moving reflecting screens), or by making a smooth contact. The direction of heat flow (but not the amount) can be defined invariantly as towards the body the proper temperature of which rises as a result of the contact. If the proper temperatures of A and B change in the same sense, the notion of temperature fails just like the notion of simultaneity has failed.

If now A and B are identical bodies, orthodox theory says that both proper observers will judge the other's temperature as low. If the usual relation exists between heat flow and temperature, it then follows that, on contact, both proper temperatures will fall. While this kind of situation is acceptable in the case of clocks (each observer sees the other clock as slow), it is not acceptable in the case of thermometers. The reason is that temperature inequality must, in any reasonable use of language, imply the possibility of heat flow, while there is nothing to correspond to this in the case of clocks. This is the most direct argument I have been able to find in favour of a Lorentz-invariant temperature, at least for the case when a temperature exists and heat transfer is at right angles to the relative motion.

(*D*) *Another self-contradictory consequence of the orthodox transformation.* Let I be the rest frame of A, in which B has velocity u in the positive x-direction. Suppose another inertial frame I' has velocity v along the positive x-direction of I. If the suffix O refers to measurements in the appropriate rest frame, the conventional theory yields

$$\text{In } I\text{: } T_A = T_{AO},\ T_B = T_{BO}/\beta(u)$$
$$\text{In } I'\text{: } T'_A = T_A/\beta(v)$$
$$T'_B = T_{BO}/\beta(u') = T_B/\beta(v)(1-uv/c^2)$$

It follows that

$$\frac{T'_A}{T'_B} = \frac{T_A}{T_B}\left(1 - \frac{uv}{c^2}\right) \qquad (6)$$

Suppose u is positive. Then "no heat flow in I (that is, $T_A = T_B$)" implies "heat flow from A to B" for a frame I' having negative v, and "heat flow from B to A" for a frame I' having positive v. Yet the physical situations "A gains heat from B" or "A loses energy to B" or "A does neither" should not depend on the inertial frame from which the bodies are observed. The orthodox relativistic theory thus uses the temperature concept in a thermodynamically quite unusual manner. This difficulty does not arise if temperature is Lorentz-invariant, as proposed earlier[6].

(*E*) *Difficulties with the kinetic definition of temperature.* It may be thought that another attack on the problem is possible by appeal to the equipartition theorem. Suppose an average is taken over all (identical) particles $i = 1, 2, \ldots, n$ of a system at a given instant. Then, if $u_{i\alpha}$, with $\alpha = x,y,z$, are typical velocity components, the theorem states that in I_0

$$\langle\!\langle m_0 u^2_{i\alpha} \rangle\!\rangle = kT_0 \qquad (7)$$

One then considers the velocity components of a typical particle as seen from I, making allowance for the bodily motion of the system by subtracting the velocity of the centre-of-mass. The random (or thermal) velocity components of a particle thus obtained are compared with the velocity components of the same particle in I_0, and each is found to be smaller by a certain factor for the y- and z- components, and by a factor β-times as big for the x-components[1,3]. One infers that a body appears to be cooler when moving, and Professor Fremlin remarks that the transformation law (2) is therefore the simplest.

A minor difficulty in this argument is that the factor is different for different velocity components, and for no component such as to support the law (2). A major difficulty can be explained by considering equation (7) as the definition of temperature for a gas of particles which can move with relativistic speeds, the system being still considered in its centre-of-mass frame. But this procedure is wrong and underestimates what is in fact the generally accepted temperature of such a system. This satisfies[8]

$$\left\langle\!\!\left\langle \frac{m_0 u^2_{i\alpha}}{\sqrt{(1-u_i^2/c^2)}} \right\rangle\!\!\right\rangle = kT_0 \qquad (8)$$

rather than equation (7). This example illustrates that in a new situation one can safely proceed from a new distribution law to a new equipartition theorem; but one cannot rely on a restricted equipartition theorem to furnish a guide as to the parameters in a new distribution law. This second and unreliable course appears to lie behind the discussion in refs. 1 and 3.

(*F*) *Generalized equipartition*. It follows that what is needed is a generalized Boltzmann distribution in I for the particles of a system of which the centre of mass has velocity $\mathbf{w}$ when viewed from I. This is easily obtained from the appendix of ref. 6 as proportional to

$$\exp\left[-\frac{\beta(w)}{kT}\,(E - wp_1)\right] \qquad (9)$$

One may take $E = c(p_1^2 + p_2^2 + p_3^2 + m_0^2c^2)^{1/2}$. Writing $\mathbf{p}_i = m_i\mathbf{u}_i$, where $\mathbf{u}_i$ is the velocity of the ith particle in I and $m_i = m_0/\sqrt{(1 - \mathbf{u}_i^2/c^2)}$ is the inertial mass of the ith particle in I, one finds after a short calculation

$$\left\langle\!\left\langle \frac{p_{i_1}^2}{m_i} - wp_{i_1} \right\rangle\!\right\rangle = \left\langle\!\left\langle \frac{p_{i_2}^2}{m_i} \right\rangle\!\right\rangle = \left\langle\!\left\langle \frac{p_{i_3}^2}{m_i} \right\rangle\!\right\rangle = \frac{kT}{\beta(w)} \qquad (10)$$

This is a generalization of equations (7) and (8).

One may apply to equations (9) and (10) the two hypotheses $T = T_0/\beta$ and $T = T_0$, as was done in the case of the Doppler effect, and one finds that both are compatible with the generalized equipartition theorem. Thus the argument from equipartition does not enable one to discriminate on theoretical grounds between accepted theory and a Lorentz-invariant temperature.

(*G*) It is well known[9] that the transformation laws of energy and momentum for a free system of particles are different from transformation laws for a confined system. It may be of interest to point out, therefore, that the statistical thermodynamics obtained in refs. 6 and 7 by using the laws for a free system are found to remain largely valid for a confined system. Indeed, K. A. Johns and I have shown that the transformation laws for a free system are identical with those for a confined system if the stresses in the container are included as part of the system. The transformation for heat is $Q = Q_0/\beta$ (this corrects a remark in ref. 7) and leads to the second law in the form $T\Delta S \geqslant \beta\Delta Q$ in frame I.

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[1] Fremlin, J. H., *Nature*, **213**, 277 (1967).
[2] Noerdlinger, P. D., *Nature*, **213**, 1117 (1967).
[3] Williams, I. P., *Nature*, **213**, 1118 (1967).
[4] Einstein, A., *Jb. Radioakt.*, **4**, 411 (1907).
[5] Rindler, W., *Special Relativity*, 15 (Oliver and Boyd, Edinburgh, 1966).
[6] Landsberg, P. T., *Nature*, **212**, 571 (1966).
[7] Landsberg, P. T., *Proc. Phys. Soc.*, **89**, 1007 (1966).
[8] Tolman, R. C., *The Principles of Statistical Mechanics*, 97 (Clarendon Press, Oxford, 1938).
[9] Pauli, W., *Theory of Relativity* (Pergamon, London, 1958). McCrea, W. H., *Relativity Physics* (Methuen, London, 1960).

Hysteresis in the Flow through an Orifice

MCVEIGH[1] has construed my statement concerning the novelty of the hysteresis effect in a circular orifice[2] rather too literally. With the exception of Grace and Lapple[3], all his references are concerned with liquid flows which do not fall within the terms of the generally accepted definition of a compressible fluid. The effect of compressibility in a gas flowing at a Mach number nearly equal to unity can never be achieved in a liquid under normal laboratory conditions. In a flowing gas it is the compressibility that is responsible for the change in the area of the jet which leads to attachment to the wall surface, the hysteresis itself being due to fluid viscosity. In a liquid, other factors such as cavitation may well be the triggering mechanism.

At the time the note on hysteresis was communicated[2], the results of experiments with liquid flows in long orifices were known but in no case was there a well defined hysteresis loop. The result reported by Grace and Lapple may well be the effect of jet attachment on the chamfered downstream face of their long orifice ($l/d = 1\cdot0$). Careful measurements of air flowing through orifices with l/d ratios between 0·60 and 2·0 and between 0·40 and 0·020 have shown that hysteresis does not occur when the downstream face is plane. The experimental points defining the hysteresis loop in the case of an orifice with an l/d ratio of 0·5 can scarcely be interpreted as a scatter as implied by McVeigh. The hysteresis loop can be reproduced with the same tolerance in measurement as over any other part of the flow range.

Details of the experimental techniques used in these investigations, and in particular the care taken to avoid uncertainties of the type mentioned by McVeigh, have recently been described[4]. It is possible that in testing similar orifices earlier investigators may have overlooked the existence of a hysteresis phenomenon because of the scatter of their measurements which, as in the report by Grace and Lapple, have been attributed to experimental limitations. The fact that the hysteresis loop can be reproduced accurately each time is evidence that it is a natural phenomenon and does not result from any random effects associated with the experimental technique.

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[1] McVeigh, J. C., *Nature*, **212**, 918 (1966).
[2] Deckker, B. E. L., *Nature*, **209**, 604 (1966).
[3] Grace, H. P., and Lapple, C. E., *Trans. Amer. Soc. Mech. Eng.*, **73**, 639 (1951).
[4] Deckker, B. E. L., and Chang, Y. F., *Thermo. and Fluid Mechanics Convention*, 1966 (Inst. Mech. Engrs. (London), Paper No. 7).

THE SOLID STATE

Conversion of Single Crystals of Chlorapatite into Single Crystals of Hydroxyapatite

SINGLE crystals of synthetic hydroxyapatite with minimum dimensions larger than about 0·1 mm have been required for X-ray and neutron diffraction investigations but they have not been available. The solid state exchange of chloride ions in single crystals of chlorapatite for hydroxyl ions, as reported here, produces suitably large single crystals of hydroxyapatite. The exchange holds additional interest both as a process and for the effects of a partial exchange on crystal properties.

The chemistry and preparation of hydroxyapatite, $Ca_{10}(PO_4)_6(OH)_2$, have been investigated extensively, partly because it is the prototype for the inorganic component of bones and teeth. Hydroxyapatite as normally prepared by precipitation from aqueous solutions, or by solid state reactions at about 1,000° C, has crystals which are at most only a few microns in size. Hydroxyapatite crystals prepared by the hydrothermal bomb method[1-4] have typical dimensions of 1 × 0·1 × 0·1 mm. Though small, they have been used for X-ray studies, but have not been used in neutron diffraction experiments. Although the bomb product is sufficiently well crystallized to be suitable for thermodynamic experiments, it has the undesirable feature that it is often contaminated by β-tricalcium phosphate and β-calcium pyrophosphate. This necessitates sorting by hand which, because of the small size of the particle, must be done under a microscope. Furthermore, apatites prepared by the hydrothermal bomb method may depart from stoichiometric composition under certain conditions[5-8].

Polycrystalline hydroxyapatite has been prepared from both mineralogical fluorapatite[9] and synthetic chlorapatite[10] powders heated in steam (1,360° C and 800° C,